2025 A/L – Combined Maths – I

Marking

Combined Maths
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Part – B

(11) (a) $x \in \mathbb{R}$

$$f(x) = x^2 - 2kx - k^2 - 1 \quad ; \quad k \in \mathbb{R}$$
$$f(x) = x^2 - 2kx - (k^2+1)$$

$x = 0$ විට,

$$f(0) = 0^2 - 2k \times 0 - (k^2+1)$$
$$f(0) = -(k^2+1)$$

$k \in \mathbb{R}$ නිසා

$$k^2 \geq 0$$

$$\therefore f(0) \neq 0$$

$\therefore$ එබැවින්, $f(x) = 0$ හි මූලයක් නොවේ.

$$f(x) = 0$$
$$x^2 - 2kx - (k^2+1) = 0$$

$$\Delta_{f(x)} = (-2k)^2 - 4 \times 1 \times (-)(k^2+1)$$
$$= 4k^2 + 4(k^2+1)$$
$$= 4(2k^2+1)$$

$\Delta_{f(x)} > 0$; $\because k \in \mathbb{R}$, $\therefore k^2 \geq 0$.

$\therefore$ $f(x) = 0$ සමීකරණයට, ශූන්‍ය නොවන තාත්වික හා ප්‍රභින්න මූල 2ක් ඇත.

$$\alpha + \beta = 2k \qquad \alpha\beta = -(k^2+1)$$

$$\gamma = \frac{1}{2\alpha} \qquad \delta = \frac{1}{2\beta}$$

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$$\gamma + s = \frac{1}{2\alpha} + \frac{1}{2\beta} \qquad \gamma s = \frac{1}{2\alpha} \times \frac{1}{2\beta}$$

$$= \frac{(\alpha+\beta)}{2\alpha\beta} \qquad = \frac{1}{4\alpha\beta}$$

$$= \frac{2k}{2(-)(k^2+1)} \qquad = \frac{1}{4(-)(k^2+1)}$$

$$= \frac{-k}{(k^2+1)} // \qquad = \frac{-1}{4(k^2+1)} //$$

γ සහ s මූල වන x වල වර්ගජ සමීකරණය,

$$x^2 - (\gamma+s)x + \gamma s = 0$$

$$x^2 - \frac{(-k)}{(k^2+1)}x + \frac{(-1)}{4(k^2+1)} = 0$$

$4(k^2+1) \times$ න්,

$$4(k^2+1)x^2 + 4kx - 1 = 0 //$$

$$(\gamma - s)^2 = \gamma^2 - 2\gamma s + s^2$$

$$= (\gamma+s)^2 - 4\gamma s$$

$$= \frac{k^2}{(k^2+1)^2} - \cancel{4}\frac{(-1)}{\cancel{4}(k^2+1)}$$

$$= \frac{k^2 + (k^2+1)}{(k^2+1)^2}$$

$$(\gamma - s)^2 = \frac{2k^2+1}{(k^2+1)^2}$$

$$\sqrt{(\gamma-s)^2} = \sqrt{\frac{2k^2+1}{(k^2+1)^2}}$$

$$|\gamma - s| = \frac{\sqrt{2k^2+1}}{|k^2+1|}$$

$$\therefore |\gamma - s| = \frac{\sqrt{2k^2+1}}{k^2+1} //$$

$\because \gamma, s \in \mathbb{R}$ හා

$k \in \mathbb{R}$ නිසා $k^2 \geq 0$

$\therefore k^2+1 > 0$

$$y = x^3 + 9x^2 + 3x + 1 \quad \text{— ①}$$
$$y = x^3 + x^2 - x + 2 \quad \text{— ②}$$

① හා ② ශ්‍රිත වල ඡේදන ලක්ෂ්‍ය වල x-ඛණ්ඩාංක ලැබෙන විට,

$$① = ②$$
$$x^3 + 9x^2 + 3x + 1 = x^3 + x^2 - x + 2$$
$$8x^2 + 4x - 1 = 0$$

ලෙස,

$$4(1^2+1)x + 4 \times 1 \times x - 1 = 0$$ ලෙස දැක්විය හැක.

$k = 1$ විට,

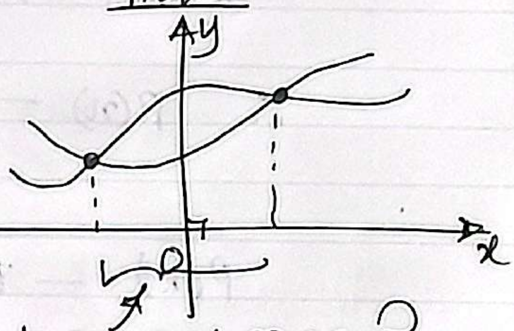
$$4(k^2+1)x + 4kx - 1 = 0 \text{ වේ.}$$

එනම් මෙම සමීකරණයේ මූල r හා s වේ.

∴ ඡේදන ලක්ෂ්‍ය 2 අතර දුර $= |r - s|$

$$= \frac{\sqrt{2k^2+1}}{k^2+1}$$
$$= \frac{\sqrt{2 \times 1^2 + 1}}{1^2+1}$$
$$= \frac{\sqrt{3}}{2} \text{ ඒකක}$$

Note



* මූල අතර වෙනසෙහි මාපාංක අගය

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Combined Maths

Combined Maths

(b) (i) $a \in \mathbb{R}$

$P(x) \longrightarrow (x-a)$ යනු සාධකයකි.

සාධක ප්‍රමේයට අනුව, (හෝ ශේෂ ප්‍රමේයට අනුව,)

$P(a) = 0$

භාජ්‍යය = භාජකය × ලබ්ධිය + ශේෂය

$$P(x) \equiv (x-a)\times Q(x) + 0$$

$$P(x) = (x-a)\times Q(x) \quad \text{——} \; ①$$

x විෂයෙන් අවකලනයෙන්,

$$P'(x) = (x-a)\times Q'(x) + Q(x)\times 1$$

$(x-a)\times$ න්,

$$(x-a)\,P'(x) = (x-a)^2\times Q'(x) + (x-a)\,Q(x) \quad \text{——} \; ②$$

① න් ② ට ආදේශයෙන්,

$$(x-a)\,P'(x) = (x-a)^2\, Q'(x) + \cancel{(x-a)}\; P(x)$$

$$\therefore \quad P(x) - (x-a)\times P'(x) = (x-a)^2 \underbrace{(-)\,Q'(x)}_{S(x)\text{ ලෙස ගනිමු.}}$$

$$P(x) - (x-a)\times P'(x) = (x-a)^2 \times S(x)$$

∴ මෙසේ වන පරිදි $S(x)$ බහුපදයක් පවතී.

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(ii) $x \in \mathbb{R}$

$$g(x) = x^3 - \lambda x^2 - 2x - (x-2)(3x^2 + \mu x - 2)$$

$(x-2)$ න් $g(x)$ බෙදූ විට ශේෂ ප්‍රමේයට අනුව,

ශේෂය $= g(2)$

සාධකයක් නිසා, $g(2) = 0$

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$$2^3 - \lambda \times 2^2 - 2\times 2 - (2-2)(3\times 2^2 + \mu \times 2 + 2) = 0$$
$$8 - 4\lambda - 4 = 0$$
$$4\lambda = 4$$
$$\lambda = 1 //$$

$g(x)$, $(x-1)$ න් බෙදූ විට ශේෂ ප්‍රමේයයට අනුව,

ශේෂය $= g(1)$

$g(1) = -3$ බව දී ඇත.

$$(+1)^3 - \lambda(1^2) - 2\times 1 - (1-2)(3\times 1^2 + \mu \times 1 - 2) = -3$$

$$1 - \lambda - 2 - (-1)(1+\mu) = -3$$
$$\cancel{1} - \cancel{\lambda} - 2 + (1+\mu) = -3$$
$$1 + \mu = -1$$
$$\mu = -2 //$$

$$\therefore g(x) = x^3 - x^2 - 2x - (x-2)(3x^2 - 2x - 2)$$

$$g(x) = (x^3 - x^2 - 2x) - (x-2)(3x^2 - 2x - 2)$$

$P(x) = x^3 - x^2 - 2x$ නම්,

$P'(x) = 3x^2 - 2x - 2$ වේ.

$$\therefore g(x) = P(x) - (x-2)P'(x) \quad \text{——} ①$$

$P(x)$ හි $x-a$ සාධකයක් විට, ඉහත සාධකයට අනුව,

$P(x) - (x-a)P'(x) = (x-a)^2 \times S(x)$ ලෙස ලිවිය හැක.

නමුත් $P(x) = x^3 - x^2 - 2x$ විට,

$x = 2$ නම්,

$P(2) = 2^3 - 2^2 - 2\times 2 = 0$

$\therefore$ $x-2$ යනු $P(x)$ හි සාධකයකි.

$\therefore$ මෙම $P(x) - (x-2)P'(x) = (x-2)^2 \times S(x)$ ලෙස ලිවිය හැක. ②

②න් ①ට ආදේශයෙන්

$$g(x) = (x-2)^2 \times S(x)$$

$\therefore$ $(x-2)^2$ යනු $g(x)$ හි සාධකයකි.

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⑫ (a) පොත් 5 ක්

ළමුන් A B C

(i)

A	B	C
2	2	1

ලැබිය හැකි විධි ගණන $= {}^5C_2 \times {}^3C_2 \times {}^1C_1$

$$= \frac{5!}{2!\,3!} \times 3 \times 1$$

$$= \frac{5 \times 4 \times \cancel{3!}}{2 \times 1 \times \cancel{3!}} \times 3$$

$$= 30$$

(ii)

A	B	C	ආකාර ගණන	
2	2	1	${}^5C_2 \times {}^3C_2 \times {}^1C_1$	$= 30$
2	1	2	${}^5C_2 \times {}^3C_1 \times {}^2C_2$	$= 30$
1	2	2	${}^5C_1 \times {}^4C_2 \times {}^2C_2$	$= 30$
3	1	1	${}^5C_3 \times {}^2C_1 \times {}^1C_1$	$= 20$
1	3	1	${}^5C_1 \times {}^4C_3 \times {}^1C_1$	$= 20$
1	1	3	${}^5C_1 \times {}^4C_1 \times {}^3C_3$	$= 20$
				150

Combined Maths

(b) $r \in \mathbb{Z}^+$ Dilshan Weerakkody

$$u_r = \frac{7r+4}{r(r+1)(r+2)}$$

$$f(r) = \frac{A}{r} \qquad g(r) = \frac{B}{r+1} \quad ; A, B \in \mathbb{R}$$

$$u_r = [f(r) - f(r+2)] + [g(r) - g(r+1)]$$

$$\frac{7r+4}{r(r+1)(r+2)} = \frac{A}{r} - \frac{A}{(r+2)} + \frac{B}{(r+1)} - \frac{B}{(r+2)}$$

$r(r+1)(r+2) \times$ න්,

$$7r + 4 = A(r+1)(r+2) - A(r)(r+1) + B(r)(r+2) - B(r)(r+1)$$

r^2 සංගු; $0 = A - A + B - B$

$0 = 0$

r^1 සංගු; $7 = 3A - A + 2B - B$

$2A + B = 7$ — ①

r^0 සංගු; $4 = 2A$ — ②

② න් $A = 2$ // . ① න්, $B = 3$ // .

$\therefore \quad f(r) = \frac{2}{r}$ // $\qquad g(r) = \frac{3}{r+1}$ //

$$\sum_{r=1}^{n} u_r = \sum_{r=1}^{n} [f(r) - f(r+2)] + \sum_{r=1}^{n} [g(r) - g(r+1)]$$

$r=1 \quad u_1 = f(1) - \cancel{f(3)} + g(1) - \cancel{g(2)}$

$r=2 \quad u_2 = f(2) - \cancel{f(4)} + \cancel{g(2)} - \cancel{g(3)}$

$r=3 \quad u_3 = \cancel{f(3)} - \cancel{f(5)} + \cancel{g(3)} - \cancel{g(4)}$

$r=4 \quad u_4 = \cancel{f(4)} - f(6) + \cancel{g(4)} - \cancel{g(5)}$

$r=5 \quad u_5 = \cancel{f(5)} - f(7) + \cancel{g(5)} - g(6)$

$\vdots$

$$\begin{array}{llllll} r = n-3 & u_{n-3} & = & f(n-3) - \cancel{f(n-1)} & + & g(n-3) - \cancel{g(n-2)} \\ r = n-2 & u_{n-2} & = & f(n-2) - \cancel{f(n)} & + & \cancel{g(n-2)} - \cancel{g(n-1)} \\ r = n-1 & u_{n-1} & = & \cancel{f(n-1)} - f(n+1) & + & \cancel{g(n-1)} - \cancel{g(n)} \\ r = n & u_n & = & \cancel{f(n)} - f(n+2) & + & \cancel{g(n)} - g(n+1) \end{array}$$

$$u_1 + u_2 + u_3 + \dots + u_n = f(1) + f(2) + g(1) - f(n+1) - f(n+2) - g(n+1)$$

$$\sum_{r=1}^{n} u_r = \frac{2}{1} + \frac{2}{2} + \frac{3}{2} - \frac{2}{(n+1)} - \frac{2}{(n+2)} - \frac{3}{(n+2)}$$

$$\sum_{r=1}^{n} u_r = \frac{9}{2} - \frac{2}{(n+1)} - \frac{5}{(n+2)} \,//$$

$$\lim_{n \to \infty} \sum_{r=1}^{n} u_r = \frac{9}{2} - 2 \underbrace{\lim_{n \to \infty} \frac{1}{n+1}}_{0} - 5 \underbrace{\lim_{n \to \infty} \frac{1}{n+2}}_{0}$$

$$\sum_{r=1}^{\infty} u_r = \frac{9}{2} \,//$$

පද ∞ යන + සංඛ(ය) වන සීමිත පරිමිතයකි. ශ්‍රේණිය අභිසාරී වේ.

පද ∞ යේ ඓක්‍යය $= 9/2 \,//$

$$\sum_{r=1}^{2n} u_r = \frac{9}{2} - \frac{2}{(2n+1)} - \frac{5}{(2n+2)}$$

$$\lim_{n \to \infty} \sum_{r=1}^{2n} u_r = \frac{9}{2} - 2 \underbrace{\lim_{n \to \infty} \frac{1}{(2n+1)}}_{0} - 5 \underbrace{\lim_{n \to \infty} \frac{1}{2n+2}}_{0}$$

$$\lim_{n \to \infty} \sum_{r=1}^{2n} u_r = \frac{9}{2} \,//$$

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$$\sum_{r=1}^{n-1} u_{n-r} = u_{n-1} + u_{n-2} + u_{n-3} + \dots + u_1$$

$$= u_1 + u_2 + u_3 + \dots + u_{n-1}$$

$$= \sum_{r=1}^{n-1} u_r$$

$$\sum_{r=1}^{n-1} u_r = \frac{9}{2} - \frac{2}{n} - \frac{5}{n+1}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n-1} u_r = \frac{9}{2} - 2\underbrace{\lim_{n\to\infty}\frac{1}{n}}_{0} + 5\underbrace{\lim_{n\to\infty}\frac{1}{n+1}}_{0}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n-1} u_r = \frac{9}{2} //$$

$$\lim_{n\to\infty}\left(\sum_{r=1}^{2n} u_r + m\sum_{r=1}^{n-1} u_{n-r}\right) = 18$$

$$\lim_{n\to\infty}\sum_{r=1}^{2n} u_r + m \times \lim_{n\to\infty}\sum_{r=1}^{n-1} u_r = 18$$

$$\frac{9}{2} + m\frac{9}{2} = 18$$

$$9 + 9m = 36$$

$$9m = 27$$

$$m = 3 //$$

Combined Maths
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(13) $A = \begin{pmatrix} 2 & a \\ 3 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 2 & 1 & a \\ 1 & 4 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 4+a & 7 \\ 6 & 7 \\ 3a & 4 \end{pmatrix}$ $\quad a \in \mathbb{R}$

A^{-1} පැවතීමට $\det A \neq 0$ වියයුතුය.

$\det A = \begin{vmatrix} 2 & a \\ 3 & 1 \end{vmatrix} = 2\times 1 - a\times 3$

$\det A = 2 - 3a$

$\det A \neq 0$

$2 - 3a \neq 0$

$a \neq 2/3$

$a \in \mathbb{R} - \{\frac{2}{3}\}$ $\quad$ Dilshan Weeralkkody

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$$AB = \begin{pmatrix} 2 & a \\ 3 & 1 \end{pmatrix}_{2\times 2} \begin{pmatrix} 2 & 1 & a \\ 1 & 4 & 1 \end{pmatrix}_{2\times 3}$$

$$AB = \begin{pmatrix} 4+a & 2+4a & 3a \\ 7 & 7 & 3a+1 \end{pmatrix}_{2\times 3} //$$

$B^T = \begin{pmatrix} 2 & 1 \\ 1 & 4 \\ a & 1 \end{pmatrix}_{3\times 2}$ $\quad$ $A^T = \begin{pmatrix} 2 & 3 \\ a & 1 \end{pmatrix}_{2\times 2}$

$$B^T A^T - C$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 4 \\ a & 1 \end{pmatrix}_{3\times 2} \times \begin{pmatrix} 2 & 3 \\ a & 1 \end{pmatrix}_{2\times 2} = \begin{pmatrix} 4+a & 7 \\ 6 & 7 \\ 3a & 4 \end{pmatrix}_{3\times 2}$$

$\circledast$ $B^T A^T = (AB)^T$ නිසා,

$$\begin{pmatrix} 4+a & 7 \\ 2+4a & 7 \\ 3a & 3a+1 \end{pmatrix}_{3\times 2} = \begin{pmatrix} 4a+a & 7 \\ 6 & 7 \\ 3a & 4 \end{pmatrix}_{3\times 2}$$

අනුරූප අවයව සමාන කිරීමෙන්,

$$2+4a = 6 \qquad\qquad 3a+1 = 4$$
$$4a = 4 \qquad\qquad 3a = 3$$
$$a = 1 \qquad\qquad a = 1$$

$$\therefore \underline{\underline{a = 1}}$$

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$$A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} \qquad A^{-1} = \frac{1}{(-1)}\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$$

$$\det A = 2 - 3\times 1 = (-1) \qquad A^{-1} = \begin{pmatrix} -1 & 1 \\ +3 & -2 \end{pmatrix}_{2\times 2} \,//$$

$$A - A^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ +3 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}_{2\times 2} + \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}_{2\times 2}$$

$$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \underline{\underline{3I}}$$

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(b) $$w = -\frac{\sqrt{2}\,(3+i)}{(1+2i)}$$

$$w = -\frac{\sqrt{2}\,(3+i)}{(1+2i)} \times \frac{(1-2i)}{(1-2i)}$$

$$w = -\frac{\sqrt{2}\,(3+i)(1-2i)}{1+4}$$

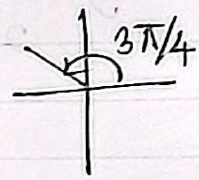
$$w = -\frac{\sqrt{2}\,(3-6i+i-2i^2)}{5}$$

$$w = -\frac{\sqrt{2}\,(5-5i)}{5}$$

$$w = -\sqrt{2}\,(1-i)$$

$$w = \sqrt{2}\,[(-1)+i]$$

$$w = \sqrt{2} \times \sqrt{(-1)^2+1^2}\,\left\{\frac{-1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right\}$$

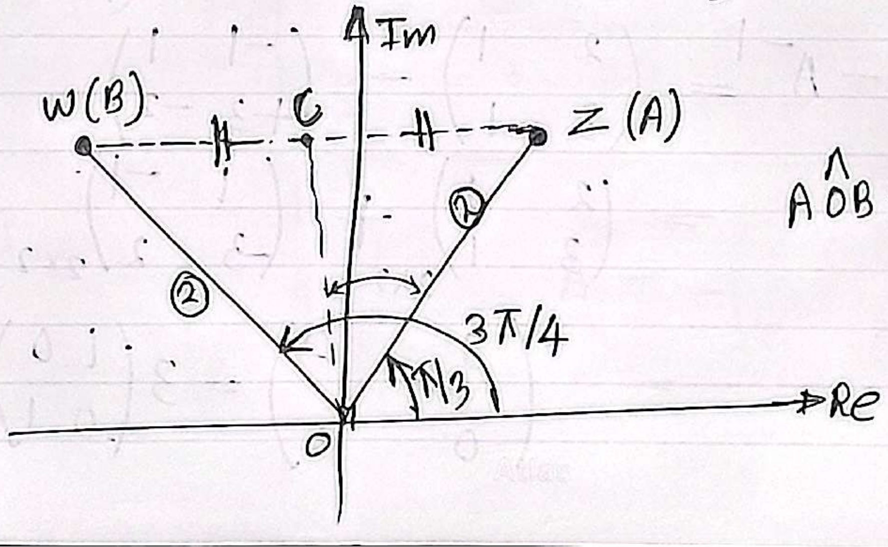


$$w = \sqrt{2} \times \sqrt{2}\,\left\{\cos 3\pi/4 + i \sin 3\pi/4\right\}$$

$$w = 2\,(\cos 3\pi/4 + i \sin 3\pi/4).$$

$\therefore |w| = 2$ // $\quad$ $\mathrm{Arg}(w) = 3\pi/4$ //

$|z| = 2 \qquad \mathrm{Arg}(z) = \pi/3 \quad ; \quad z \in \mathbb{C}$



$$A\hat{O}B = \frac{3\pi}{4} - \frac{\pi}{3}$$
$$= \frac{5\pi}{12}$$
$$(75°)$$

AOB Δ ට cos නීතියෙන්,

$$\cos(A\hat{O}B) = \frac{OA^2 + OB^2 - AB^2}{2OA \cdot OB}$$

$$\cos 75^\circ = \frac{2^2 + 2^2 - AB^2}{2 \times 2 \times 2}$$

$$\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

$$\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{8 - AB^2}{8}$$

$$\frac{8(\sqrt{3}-1)}{2\sqrt{2}} = 8 - AB^2$$

$$\frac{4(\sqrt{3}-1)}{\sqrt{2}} = 8 - AB^2$$

$$\frac{4(\sqrt{3}-1)}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 8 - AB^2$$

$$\frac{4\sqrt{2}(\sqrt{3}-1)}{2} = 8 - AB^2$$

$$2\sqrt{2}(\sqrt{3}-1) = 8 - AB^2$$

$$2\sqrt{6} - 2\sqrt{2} = 8 - AB^2$$

$$AB^2 = 8 + 2\sqrt{2} - 2\sqrt{6}$$

Dilshan Weerakkody

AB හි මධ්‍ය ලක්ෂ්‍යය C වේ. AOB සම ද්විපාද වේ.

∴ OC යන්න ABට ලම්බ වේ. ∴ AOC සෘජුකෝණික Δයකි.

$$A\hat{O}C = \frac{5\pi}{12} \times \frac{1}{2} = \frac{5\pi}{24}$$

$$\sin A\hat{O}C = \frac{AC}{OA} = \frac{1}{2}\frac{AB}{OA}$$

$$\sin^2 A\hat{O}C = \frac{1}{4}\frac{AB^2}{OA^2} = \frac{1}{4} \times \frac{1}{4}(8 + 2\sqrt{2} - 2\sqrt{6})$$

$$\therefore \sin^2 \frac{5\pi}{24} = \frac{1}{8}(4 + \sqrt{2} - \sqrt{6})$$

Dilshan Weerakkody

(c) $m \in \mathbb{Z}^+$:

$$(1+\sqrt{3}i)^{3m}(1+i)^8 = 2^{3m+4}$$

$$1+\sqrt{3}i = \sqrt{1^2+\sqrt{3}^2}\left[\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]$$

$$1+\sqrt{3}i = 2\left(\cos \pi/3 + i\sin \pi/3\right)$$

$$(1+\sqrt{3}i)^{3m} = 2^{3m}\left[\cos m\pi + i\underbrace{\sin m\pi}_{0}\right] \quad \text{—— (1)}$$

$$(1+i) = \sqrt{1^2+1^2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$$

$$(1+i) = \sqrt{2}\left[\cos \pi/4 + i\sin \pi/4\right]$$

$$(1+i)^8 = \sqrt{2}^8\left[\underbrace{\cos 2\pi}_{1} + i\underbrace{\sin 2\pi}_{0}\right]$$

$$(1+i)^8 = 2^4 \quad \text{—— (2)}$$

$$(1+\sqrt{3}i)^{3m} \times (1+i)^8 = 2^{3m+4}$$

$$2^{3m}\left[\cos m\pi\right] \times 2^4 = 2^{3m+4}$$

$$2^{3m+4} \times (\cos m\pi) = 2^{3m+4}$$

$m \in \mathbb{Z}^+$ නිසා,

$$\cos m\pi = 1$$

$$\cos m\pi = \cos 0$$

$$m\pi = 2n\pi \pm 0$$

$$m\pi = 2n\pi$$

$m = 2n$; $n \in \mathbb{Z}^+$ වේ.

$m_{අවම} = 2 \times 1 = 2$ // ($\because n_{අවම}$)

(14) a) $x \in \mathbb{R} - \{1\}$

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$$y = f(x) = \frac{x^2 + x + p}{(x-1)^2} \quad ; \quad p \in \mathbb{R}$$

තිරස් ස්පර්ශෝන්මුඛ සඳහා,

$$\lim_{x \to \pm\infty} y = \lim_{x \to \pm\infty} \frac{x^2 + x + p}{(x-1)^2}$$

$$= \lim_{x \to \pm\infty} \frac{1 + 1/x + p/x^2}{(1 - 1/x)^2}$$

$$y = 1 //$$

$y = 1$ දී තිරස් ස්පර්ශෝන්මුඛයක් පවතී.

$y = f(x)$ හා $y = 1$ වක්‍ර $x = -1/3$ දී ඡේදනය වන බව දී ඇත.

$y = f(x)$ හා $y = 1$ ඡේදනය වන විට,

$$f(x) = 1$$

$$\frac{x^2 + x + p}{(x-1)^2} = 1$$

$$x^2 + x + p = (x-1)^2$$

$x = -1/3$ ආදේශ කරමින්,

$$\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right) + p = \left(-\frac{1}{3} - 1\right)^2$$

$$\frac{1}{9} - \frac{1}{3} + p = \frac{\cancel{4}16}{9}$$

$$1 - 3 + 9p = \cancel{4}16$$

$$9p = 18$$

$$p = 2 //$$

Combined Maths

$$\therefore f(x) = \frac{x^2 + x + 2}{(x-1)^2}$$

x විෂයෙන් අවකලනයෙන්,

$$f'(x) = \frac{(x-1)^2 \times (2x+1) - (x^2+x+2) \times 2(x-1)^1}{[(x-1)^2]^2}$$

$$f'(x) = \frac{(x-1)^2(2x+1) - 2(x-1)(x^2+x+2)}{(x-1)^4}$$

$x \neq 1$ නිසා,

$$f'(x) = \frac{(x-1)(2x+1) - 2(x^2+x+2)}{(x-1)^3}$$

$$= \frac{\cancel{2x^2} - x - 1 - \cancel{2x^2} - 2x - 4}{(x-1)^3}$$

$$= \frac{-3x-5}{(x-1)^3}$$

$$f'(x) = -\frac{(3x+5)}{(x-1)^3}$$

සිරස් ස්පර්ශෝන්මුඛ වලදී, $x - 1 = 0$

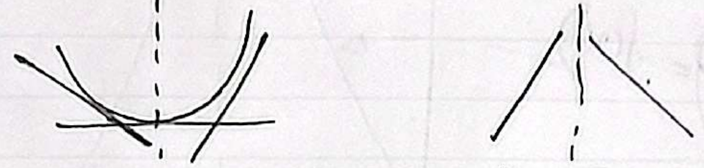
$x = 1$

ස්ථාවර ලක්ෂ්‍ය වලදී; $f'(x) = 0$

$3x + 5 = 0$

$x = -5/3$

	$x<-5/3$	$-5/3$	$-\frac{5}{3}<x<1$	1	$x>1$
$f'(x)$	$(-)$		$(+)$		$(-)$



$x = -5/3$ විට ස්ථානීය අවමයක් පවතී.

අවම ලක්ෂ්‍යය $\left(\frac{-5}{3}, \frac{7}{16}\right)$

$$y = \frac{\left(\frac{-5}{3}\right)^2 - \left(\frac{5}{3}\right) + 2}{\left(\frac{-5}{3} - 1\right)^2}$$

$$y = \frac{\frac{25}{9} - \frac{5}{3} + 2}{\frac{64}{9}}$$

$$y = \frac{25 - 15 + 18}{64}$$

$$y = \frac{28}{64} = \frac{7}{16}$$

$x < -5/3$ ශ්‍රිතය අඩුවේ

$-5/3 < x < 1$ ශ්‍රිතය වැඩිවේ.

$x > 1$ ශ්‍රිතය අඩුවේ.

<u>අක්ෂ කැපීම</u>

x අක්ෂය කපන විට,

$y = 0$

$x^2 + x + 2 = 0$

$\Delta = 1^2 - 4 \times 1 \times 2$

$\Delta < 0$

x අක්ෂය නොකපයි.

y අක්ෂය කපන විට,

$x = 0$

$y = \frac{2}{(-1)^2} = 2$

නතිවර්තන ලක්ෂ්‍ය වලදී, $f''(x) = 0$ $\quad \because f''(x) = \frac{6(x+3)}{(x-1)^4}$

$6(x+3) = 0$

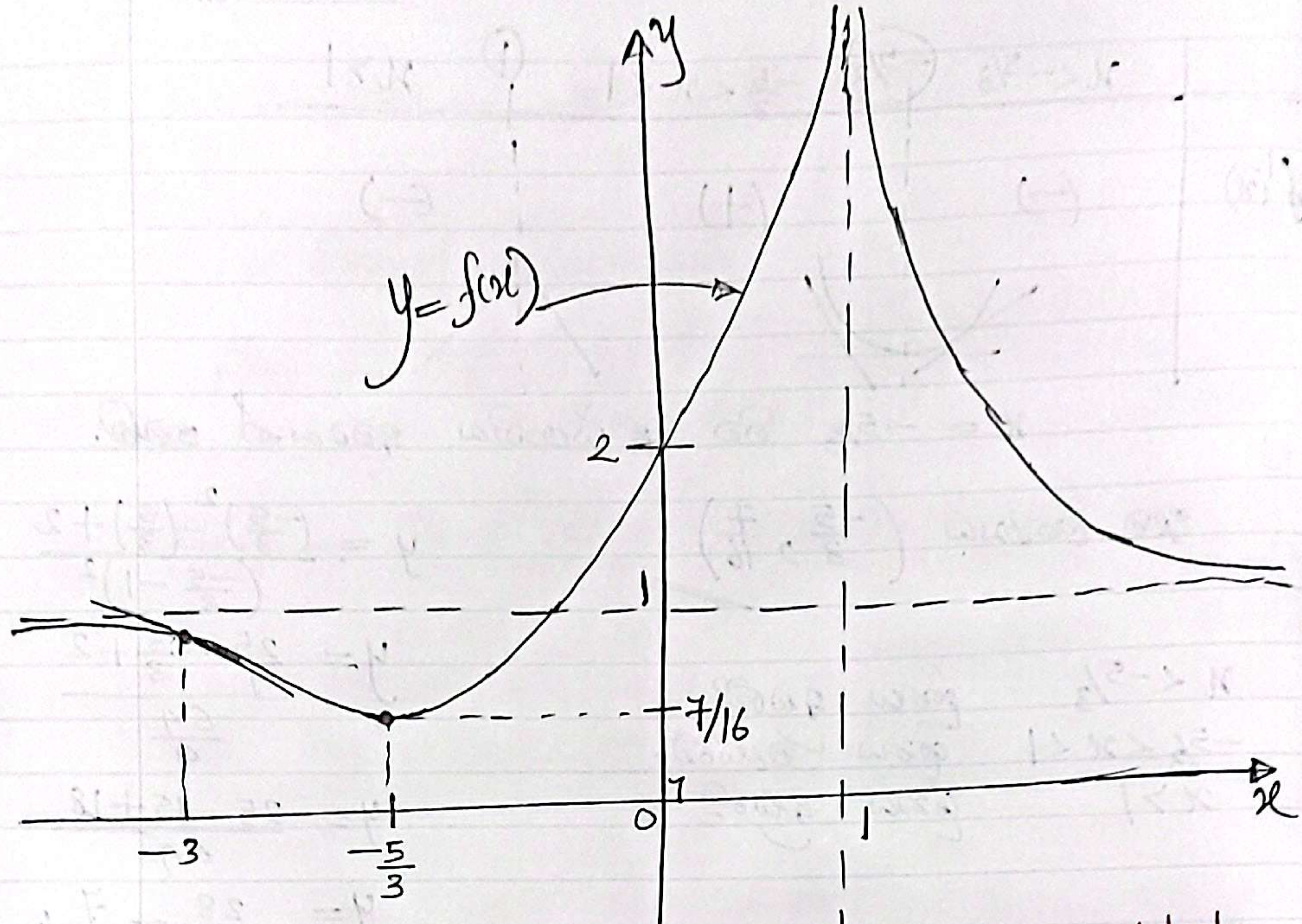
$x = -3$

		-3		1	
$f''(x)$	$(-)$		$(+)$		$(+)$
	යටි අවතල		උඩු අවතල		උඩු අවතල

$\therefore x = -3$ දී නතිවර්තන ලක්ෂ්‍යයක් ඇත.

$$y = \frac{(-3)^2 + (-3) + 2}{(-3-1)^2} = \frac{1}{2}$$

නතිවර්තන ලක්ෂ්‍යය $\left(-3, \frac{1}{2}\right)$



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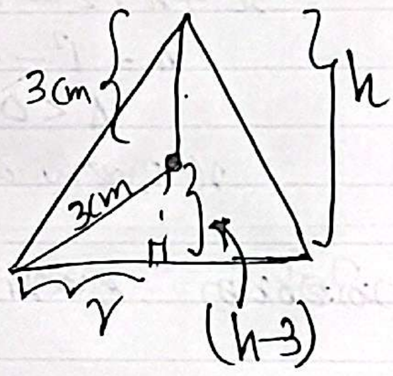
(b) $V = \frac{1}{3}\pi r^2 h$; $r \Rightarrow$ කේතුවේ පතුලේ අරය.

පයිතගරස් ප්‍රමේයයට අනුව,



$$3^2 = r^2 + (h-3)^2$$

$$9 = r^2 + (h-3)^2$$

$$\therefore\ r^2 = 9 - (h-3)^2$$

$$r^2 = 9 - (h^2 - 6h + 9)$$

$$r^2 = 6h - h^2 \,//$$

$$\therefore\ V = \frac{1}{3}\pi (6h - h^2) h$$

$$V = \frac{\pi}{3}(6h^2 - h^3) \,//$$

h විෂයෙන් අවකලනයෙන්,

$$\frac{dV}{dh} = \frac{\pi}{3}\left(6 \times 2h - 3h^2\right)$$

$$\frac{dV}{dh} = \pi\left(4h - h^2\right)$$

$$\frac{dV}{dh} = \pi h(4 - h)$$

V උපරිම හෝ අවම විට, $\frac{dV}{dh} = 0$

$$\pi h(4-h) = 0$$

$h = 0$ හෝ $h = 4$

$h \neq 0$

$h = 4$

$\frac{dV}{dh} > 0$ | $\frac{dV}{dh} < 0$

$\therefore$ $h = 4$ විට, උපරිම පරිමාව ලැබේ.

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(15) (a) $K \in \mathbb{R}$

$$I = \int \frac{\sqrt{x}}{(1-K^2x)}\,dx$$

$x = t^2$ ලෙස ගනිමු.

$$\frac{dx}{dt} = 2t$$

$$I = \int \frac{\sqrt{t^2}}{1-K^2t^2} \times 2t \; dt$$

$$I = \int \frac{2t^2}{1-K^2t^2}\,dt$$

$$I = 2\int \frac{t^2}{1-k^2t^2}\,dt$$

$$I = \frac{2}{-k^2}\int \frac{-k^2t^2}{1-k^2t^2}\,dt$$

$$I = \frac{-2}{k^2}\int \frac{(1-k^2t^2)-1}{(1-k^2t^2)}\,dt$$

$$\frac{k^2}{-2}\times I = \int 1\,dt - \int \frac{1}{1-k^2t^2}\,dt$$

$$-\frac{k^2}{2} I = t - \int \frac{1}{(1-kt)(1+kt)}\,dt$$

$$= t - \int \frac{1}{2}\times\frac{[(1+kt)+(1-kt)]}{(1-kt)(1+kt)}\,dt$$

$$= t - \frac{1}{2}\int\left(\frac{1}{1-kt} + \frac{1}{1+kt}\right)dt$$

$$-\frac{k^2}{2} I = t - \frac{1}{2}\int \frac{1}{1-kt}\,dt - \frac{1}{2}\int \frac{1}{1+kt}\,dt$$

$$= t - \frac{1}{2(-k)}\ln|1-kt| - \frac{1}{2}\times\frac{1}{k}\ln|1+kt|$$

$$-\frac{k^2}{2} I = \sqrt{x} + \frac{1}{2k}\ln|1-k\sqrt{x}| - \frac{1}{2k}\ln|1+k\sqrt{x}|$$

$$I = \frac{-2}{k^2}\sqrt{x} - \frac{1}{k^3}\ln|1-k\sqrt{x}| + \frac{1}{k^3}\ln|1+k\sqrt{x}| + C$$

C යනු අනුකල නියතයකි.

Dilshan Weerakkody

(b) $0 < x < \frac{\pi}{4}$ සඳහා, Combined Maths

$$\frac{d}{dx}\left\{\ln\left(\frac{1+\sin 2x}{\cos 2x}\right)\right\}$$

$$= \frac{1}{\left(\frac{1+\sin 2x}{\cos 2x}\right)} \times \frac{[\cos 2x \times \cos 2x \times 2 - (1+\sin 2x) \times (-\sin 2x) \times 2]}{\cos^2 2x}$$

$$= \frac{\cancel{\cos 2x}}{(1+\sin 2x)} \times \frac{[2\cos^2 2x + 2\sin 2x(1+\sin 2x)]}{\cos^{\cancel{2}} 2x}$$

$$= \frac{2[\cos^2 2x + \sin^2 2x + \sin 2x]}{(1+\sin 2x)\cos 2x}$$

$$= \frac{2\cancel{[1+\sin 2x]}}{\cancel{(1+\sin 2x)}\cos 2x} \quad ; \quad \because 0 < x < \frac{\pi}{4}$$

$$= \frac{2}{\cos 2x}$$

$$I = \int \underbrace{(\cos 2x)}_{\frac{dv}{dx}} \times \underbrace{\ln\left(\frac{1+\sin 2x}{\cos 2x}\right)}_{u} dx$$

$$I = \ln\left(\frac{1+\sin 2x}{\cos 2x}\right) \times \frac{\sin 2x}{2} - \int \frac{\sin 2x}{\cancel{2}} \times \frac{\cancel{2}}{\cos 2x}\, dx$$

$$I = \ln\left(\frac{1+\sin 2x}{\cos 2x}\right) \times \frac{\sin 2x}{2} - \int \tan 2x\, dx$$

$$I = \frac{\sin 2x}{2}\ln\left(\frac{1+\sin 2x}{\cos 2x}\right) + \frac{1}{2}\int \frac{-2\sin 2x}{\cos 2x}\, dx$$

$$I = \frac{\sin 2x}{2}\ln\left(\frac{1+\sin 2x}{\cos 2x}\right) + \frac{1}{2}\ln|\cos 2x| + C$$

C අභිමත නියතයකි.

(c) a නියතයක් $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$\int_0^{\pi/6} \frac{\cos(x+\pi/3)}{\sin x + \cos(x+\pi/3)}\,dx = \int_0^{\pi/6} \frac{\cos[(\pi/6 - x) + \pi/3]}{\sin(\pi/6 - x) + \cos[(\pi/6 - x) + \frac{\pi}{3}]}\,dx$$

$$= \int_0^{\pi/6} \frac{\cos(\pi/2 - x)}{\sin[\frac{\pi}{2} - (x+\pi/3)] + \cos(\pi/2 - x)}\,dx$$

$$= \int_0^{\pi/6} \frac{\sin x}{\cos(x+\pi/3) + \sin x}\,dx$$

$$I = \int_0^{\pi/6} \frac{\cos(x+\pi/3)}{\sin x + \cos(x+\pi/3)}\,dx \quad \text{—— (1)}$$

$$I = \int_0^{\pi/6} \frac{\sin x}{\sin x + \cos(x+\pi/3)}\,dx \quad \text{—— (2)}$$

(1) + (2) :

$$2I = \int_0^{\pi/6} \frac{\cos(x+\pi/3)}{\sin x + \cos(x+\pi/3)}\,dx + \int_0^{\pi/6} \frac{\sin x}{\sin x + \cos(x+\pi/3)}\,dx$$

$$2I = \int_0^{\pi/6} \frac{\cos(x+\pi/6) + \sin x}{\sin x + \cos(x+\pi/6)}\,dx$$

$$2I = \int_0^{\pi/6} 1\,dx$$

$$2I = [x]_0^{\pi/6}$$

$$2I = \frac{\pi}{6} - 0$$

$$\therefore I = \frac{\pi}{12}$$

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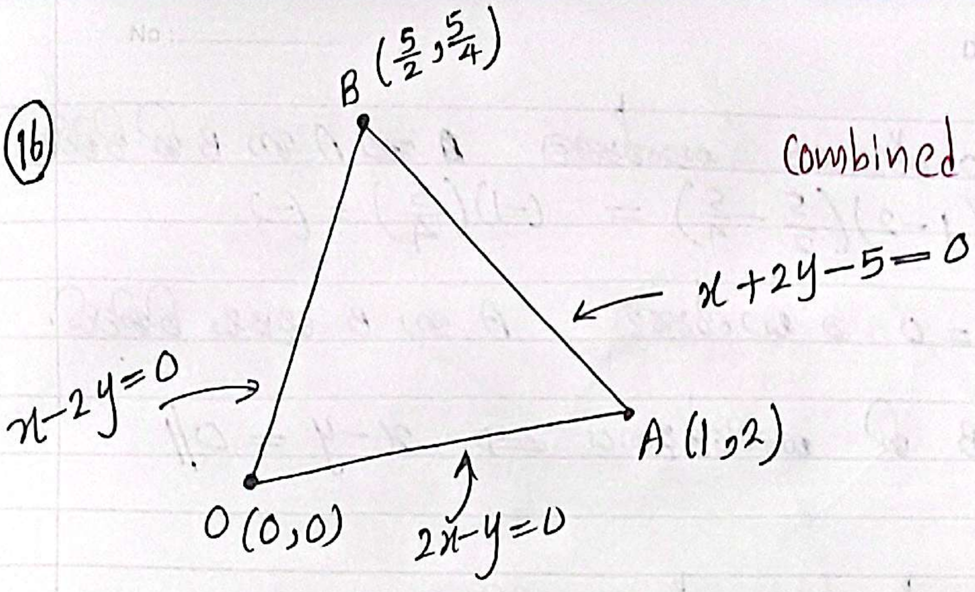
16

Combined Maths



OA අනු: $= \dfrac{2-0}{1-0} = 2$ //

OB අනු: $= \dfrac{5/4-0}{5/2-0} = \dfrac{1}{2}$ //

OA සමීකරණය,

$y = 2x$

$2x - y = 0$ //

OB සමීකරණය,

$y = \frac{1}{2}x$

$x - 2y = 0$ //

AB අනු: $= \dfrac{\frac{5}{4}-2}{\frac{5}{2}-1} = \dfrac{5-8}{10-4} = \dfrac{-3}{6} = \dfrac{-1}{2}$ //

AB සමීකරණය,

$y - 2 = -\frac{1}{2}(x-1)$

$2y - 4 = -x + 1$

$x + 2y - 5 = 0$ //

* AÔB කෝණ සමච්ඡේදකය සෙවීම,

$$\frac{2x-y}{\sqrt{2^2+(-1)^2}} = \pm\frac{(x-2y)}{\sqrt{1^2+(-2)^2}}$$

$2x - y = \pm(x - 2y)$

+/ $2x - y = x - 2y$

$x + y = 0$

−/ $2x - y = -x + 2y$

$3x - 3y = 0$

$x - y = 0$

$x-y=0$ රේඛාවට සාපේක්ෂව A හා B හි පිහිටීම:

$$(1-2)\left(\frac{5}{2}-\frac{5}{4}\right) = (-1)\left(\frac{5}{4}\right) = (-)$$

$x-y=0$ ට සාපේක්ෂව A හා B දෙපස පිහිටයි.

$\therefore$ $A\hat{O}B$ හි සමච්ඡේදකය $\Longrightarrow x-y=0$ //

(*) $O\hat{A}B$ කෝණ සමච්ඡේදකය සෙවීම.

$$\frac{x+2y-5}{\sqrt{1^2+2^2}} = \pm\frac{(2x-y)}{\sqrt{2^2+(-1)^2}}$$

$$x+2y-5 = \pm(2x-y)$$

$+/$ $x+2y-5 = 2x-y$

$x-3y+5=0$

$-/$ $x+2y-5=-2x+y$

$3x+y-5=0$

$3x+y-5=0$ රේඛාවට සාපේක්ෂව O හා B හි පිහිටීම

$$(3\times 0+0-5)\left(3\times\frac{5}{2}+\frac{5}{4}-5\right) = (-5)\left(\frac{15}{4}\right) = (-)$$

$\therefore$ $3x+y-5=0$ රේඛාවට සාපේක්ෂව O හා B දෙපස පිහිටයි.

$\therefore$ $O\hat{A}B$ හි සමච්ඡේදකය $\Longrightarrow 3x+y-5=0$ //

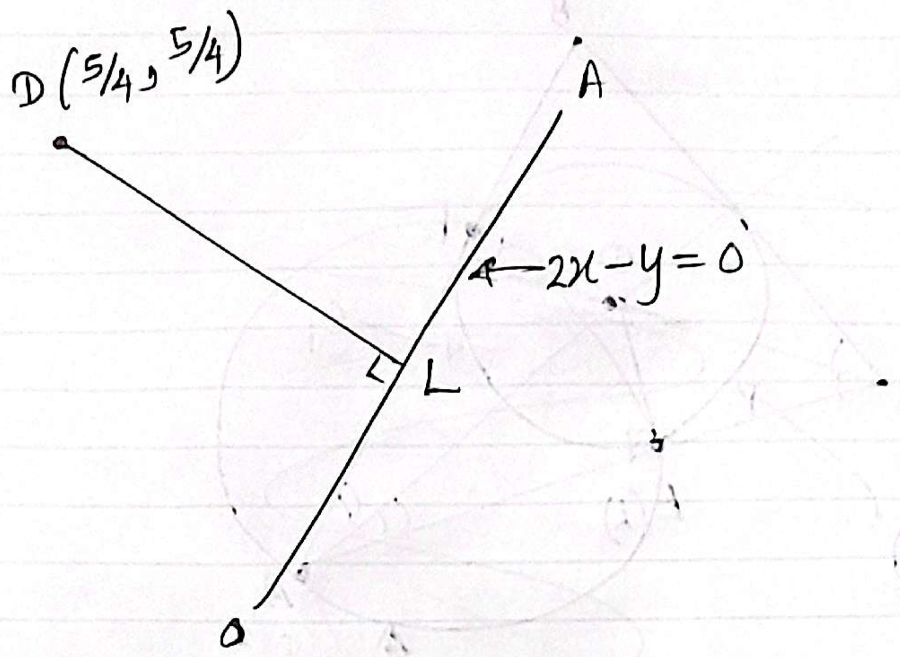
(*) ඡේදන ලක්ෂ්‍යය සෙවීම,

$$x-y=0$$
$$y=x \quad \text{—} \ ①$$
$$3x+y-5=0 \quad \text{—} \ ②$$

① න් ② ට ආදේශයෙන්

$$3x+x-5=0$$
$$x=5/4 \ //$$
$$y=5/4 \ //$$

$\therefore$ ඒවා $D \equiv \left(\frac{5}{4},\frac{5}{4}\right)$ හිදී ඡේදනය වේ.



⊛ D සිට OA ට ලම්බ දුර = DL

$$DL = \frac{\left|2\times\frac{5}{4} - \frac{5}{4}\right|}{\sqrt{2^2+(-1)^2}}$$

$$DL = \frac{\left|\frac{5}{2} - \frac{5}{4}\right|}{\sqrt{5}}$$

$$DL = \frac{\frac{5}{4}}{\sqrt{5}}$$

$$DL = \frac{\sqrt{5}}{4} \text{ ඒකක}$$

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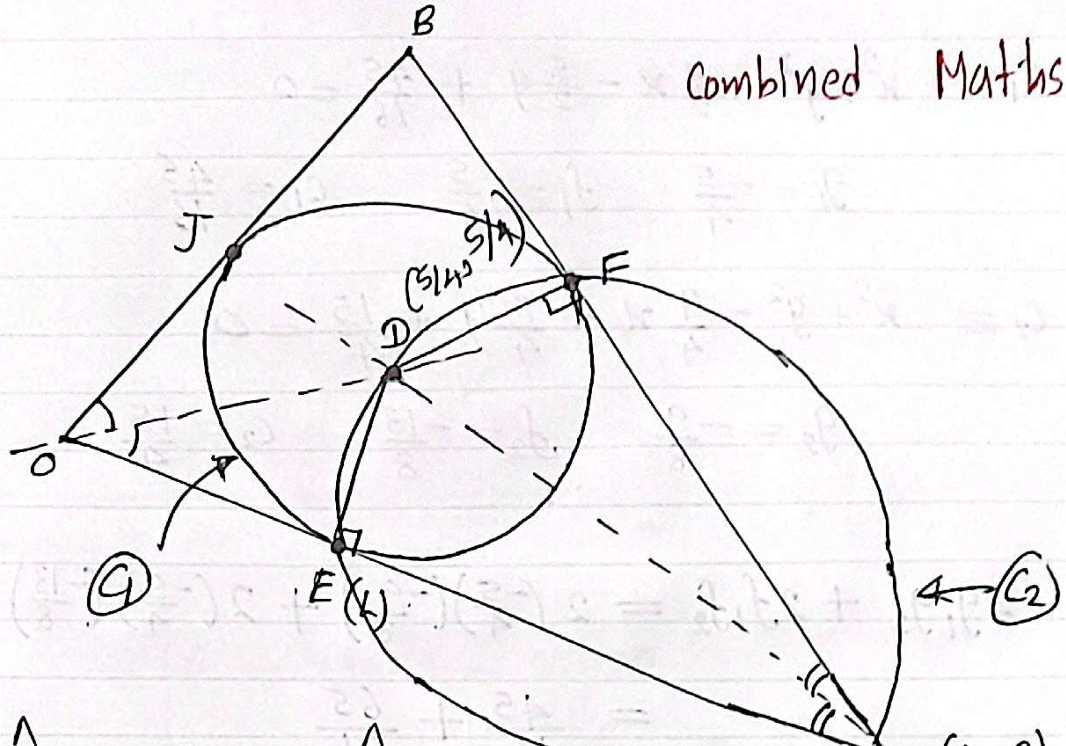
⊛ OAB Δ යේ පාද 3ට ස්පර්ශ කරන වෘත්තය, එහි අන්තර් වෘත්තය වේ. එහි කේන්ද්‍රය කෝණ සමච්ඡේදක වල ඡේදන ලක්ෂ්‍යය වේ.

∴ කේන්ද්‍රය D වූ අරය DL වූ වෘත්තය C_1 වේ.

$$C_1 \Rightarrow \left(x-\frac{5}{4}\right)^2 + \left(y-\frac{5}{4}\right)^2 = \left(\frac{\sqrt{5}}{4}\right)^2$$

$$x^2+y^2-\frac{5}{2}x-\frac{5}{2}y+\frac{45}{16}=0$$

Combined Maths



$A(1, 2)$

$D\hat{E}A = \frac{\pi}{2}$ හා $D\hat{F}A = \pi/2$ වේ.

එනම් $D\hat{E}A + D\hat{F}A = \pi$ (AEDF චතුරස්‍රයේ සම්මුඛ කෝණ 2ක එකතුව 180° වී ඇත.)

$\therefore$ AEDF යනු වෘත්ත චතුරස්‍රයකි.

එනම්, A, E, F හරහා යන වෘත්තය D හරහාද යයි.

තවද AD, C_2 හි විෂ්කම්භයක් වේ. ($\because$ $O\hat{A}B$ සමච්ඡේදකය AD වේ.

$\therefore$ C_2 යනු AD විෂ්කම්භය වන වෘත්තය වේ

$C_2 \Rightarrow \quad (x-1)(x-5/4) + (y-2)(y-5/4) = 0$

$$x^2 + y^2 - \frac{9}{4}x - \frac{13}{4}y + \frac{15}{4} = 0$$

$$\therefore \quad 4x^2 + 4y^2 - 9x - 13y + 15 = 0 \,//$$

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$$C_1 \equiv x^2+y^2-\frac{5}{2}x-\frac{5}{2}y+\frac{45}{16}=0$$

$$g_1=-\frac{5}{4} \qquad f_1=-\frac{5}{4} \qquad c_1=\frac{45}{16}$$

$$C_2 \equiv x^2+y^2-\frac{9}{4}x-\frac{13}{4}y+\frac{15}{4}=0$$

$$g_2=-\frac{9}{8} \qquad f_2=-\frac{13}{8} \qquad c_2=\frac{15}{4}$$

$$2g_1g_2+2f_1f_2 = 2\left(-\frac{5}{4}\right)\left(-\frac{9}{8}\right)+2\left(-\frac{5}{4}\right)\left(-\frac{13}{8}\right)$$

$$=\frac{45}{16}+\frac{65}{16}$$

$$=\frac{110}{16}\,//$$

$$c_1+c_2=\frac{45}{16}+\frac{15}{4}$$

$$=\frac{45}{16}+\frac{60}{16}$$

$$=\frac{105}{16}\,//$$

$2g_1g_2+2f_1f_2 \neq c_1+c_2$ වේ.

$\therefore$ C_1 හා C_2 වෘත්ත ප්‍රලම්භ නොවේ.

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Combined Maths

(17)(a) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$A \Rightarrow \theta$ හා $B \Rightarrow \theta$ යෙදීම.

$\sin(\theta+\theta) = \sin\theta \cdot \cos\theta + \cos\theta \cdot \sin\theta$

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$ //

$0 < \theta < \frac{\pi}{2}$.

$\cot\theta - 2\tan\theta = \sin 2\theta$

$\frac{\cos\theta}{\sin\theta} - \frac{2\sin\theta}{\cos\theta} = 2\sin\theta \cdot \cos\theta$

$\sin\theta \cdot \cos\theta$ x කී.

$\cos^2\theta - 2\sin^2\theta = 2\sin^2\theta \cdot \cos^2\theta$
$\cos^2\theta - 2(1-\cos^2\theta) = 2(1-\cos^2\theta) \cdot \cos^2\theta$
$\cos^2\theta - 2 + 2\cos^2\theta = 2\cos^2\theta - 2\cos^4\theta$

$2\cos^4\theta + \cos^2\theta - 2 = 0$

$a = 2$ //, $b = 1$ //, $c = -2$ //

$\cos^2\theta = t$ ලෙස ගනිමු.

$2t^2 + t - 2 = 0$
$t^2 + \frac{1}{2}t = 1$

$\left(t+\frac{1}{4}\right)^2 = 1 + \frac{1}{16}$

$\left(t+\frac{1}{4}\right)^2 = \frac{17}{16}$

$\sqrt{\left(t+\frac{1}{4}\right)^2} = \sqrt{\frac{17}{16}}$

Combined Maths

$$\left|t+\frac{1}{4}\right| = \frac{\sqrt{17}}{4}$$

$0< \theta < \pi/_2$ නිසා,

$0 < \cos\theta < 1$

$\cos^2\theta > 0$

$\therefore\ t > 0$

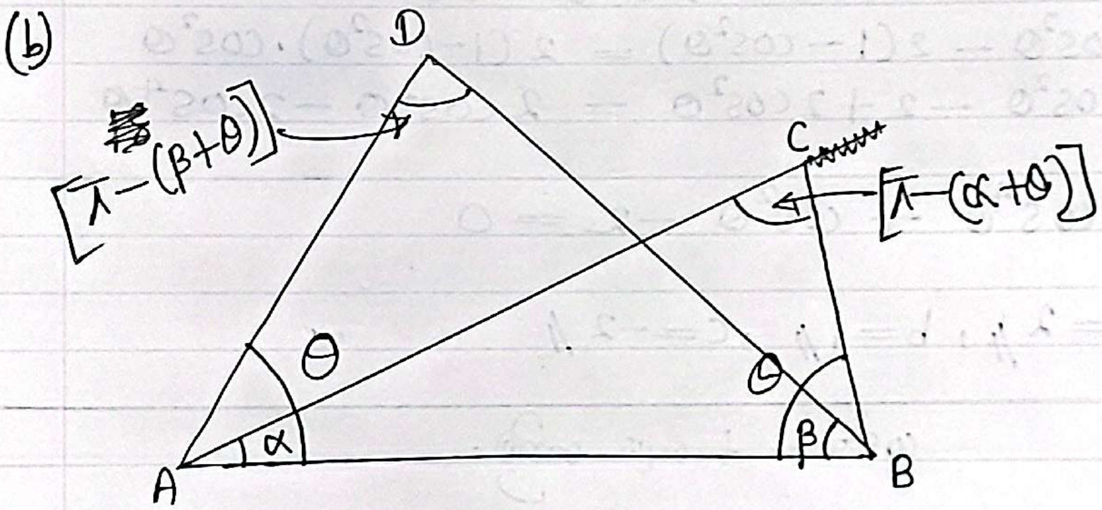
$$t+\frac{1}{4} = \frac{\sqrt{17}}{4}$$

$$t = \frac{\sqrt{17}-1}{4}$$

$$\cos^2\theta = \frac{\sqrt{17}-1}{4}$$

$$\cos\theta = \sqrt{\frac{\sqrt{17}-1}{4}}\ ;\ \because 0<\theta<\frac{\pi}{2}$$

(b)



$3AD = 4BC$

ABC Δ ට sin නීතියෙන්,

$$\frac{AB}{\sin[\pi-(\alpha+\theta)]} = \frac{BC}{\sin\alpha}$$

$$\frac{AB}{BC} = \frac{\sin(\alpha+\theta)}{\sin\alpha} \quad \text{—— (1)}$$

ABD Δ ට sin නීතියෙන්,

$$\frac{AB}{\sin[\pi-(\theta+\beta)]} = \frac{AD}{\sin\beta}$$

$$\frac{AB}{AD} = \frac{\sin(\theta+\beta)}{\sin\beta} \quad \text{--- (2)}$$

(1) සහ (2) න්,

$$BC\,\frac{\sin(\alpha+\theta)}{\sin\alpha} = AD\,\frac{\sin(\theta+\beta)}{\sin\beta}$$

$$\frac{BC}{AD} = \frac{\sin\alpha\cdot\sin(\theta+\beta)}{\sin\beta\cdot\sin(\theta+\alpha)} \;//$$

~~B~~. $3AD = 4BC$ නිසා,

$$\frac{BC}{AD} = \frac{3}{4}$$

$$\therefore \frac{\sin\alpha\cdot\sin(\theta+\beta)}{\sin\beta\cdot\sin(\theta+\alpha)} = \frac{3}{4}$$

$$4\sin\alpha\,(\sin\theta\cdot\cos\beta+\cos\theta\cdot\sin\beta) = 3\sin\beta\cdot(\sin\theta\cdot\cos\alpha + \cos\theta\cdot\sin\alpha)$$

$\sin\alpha\cdot\sin\beta\cdot\sin\theta$ ÷ න්

$$4(\cot\beta+\cot\theta) = 3(\cot\alpha+\cot\theta)$$

$$4\cot\beta + 4\cot\theta = 3\cot\alpha + 3\cot\theta$$

$$\cot\theta = \underline{\underline{3\cot\alpha - 4\cot\beta}}$$

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(c) $\underbrace{\sin^{-1}\sqrt{x}}_{\alpha} = \underbrace{\cos^{-1}\sqrt{y}}_{\beta}$

$\alpha = \sin^{-1}\sqrt{x}$ $\qquad$ $\beta = \cos^{-1}\sqrt{y}$

$\sin\alpha = \sqrt{x}$ $\qquad$ $\cos\beta = \sqrt{y}$

$$\alpha = \beta$$
$$\sin\alpha = \sin\beta$$
$$\sqrt{x} = \sqrt{1-\cos^2\beta}$$
$$\sqrt{x} = \sqrt{1-y}$$
$$x = 1-y$$
$$y = 1-x \quad \text{——— (1)}$$

$\tan[\underbrace{\tan^{-1}3x}_{\gamma} - \underbrace{\tan^{-1}2y}_{\delta}] + \tan[\underbrace{\tan^{-1}3y}_{\theta} - \underbrace{\tan^{-1}2x}_{\phi}] = 1$

$\gamma = \tan^{-1}3x$ $\qquad$ $\delta = \tan^{-1}2y$ $\qquad$ $\theta = \tan^{-1}3y$ $\qquad$ $\phi = \tan^{-1}2x$

$\tan\gamma = 3x$ $\qquad$ $\tan\delta = 2y$ $\qquad$ $\tan\theta = 3y$ $\qquad$ $\tan\phi = 2x$

$$\tan(\gamma-\delta) + \tan(\theta-\phi) = 1$$

$$\frac{\tan\gamma - \tan\delta}{1+\tan\gamma\tan\delta} + \frac{\tan\theta - \tan\phi}{1+\tan\theta\cdot\tan\phi} = 1$$

$$\frac{3x-2y}{1+3x\times 2y} + \frac{3y-2x}{1+3y\times 2x} = 1$$

$$3x-2y+3y-2x = 1+6xy$$
$$x+y = 1+6xy \quad \text{——— (2)}$$

② න් ① ට ආදේශයෙන්

$$x + (1-x) = 1 + 6x(1-x)$$

$$\cancel{x} + 1 - \cancel{x} = 1 + 6x - 6x^2$$

$$6x^2 - 6x = 0$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$x = 0$ හෝ $x = 1$ //

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